UNIT-1:VECTORS

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Created by FİZİKBAZ for students preparing for AP Physics-1 exams.

1. Frame of Reference:

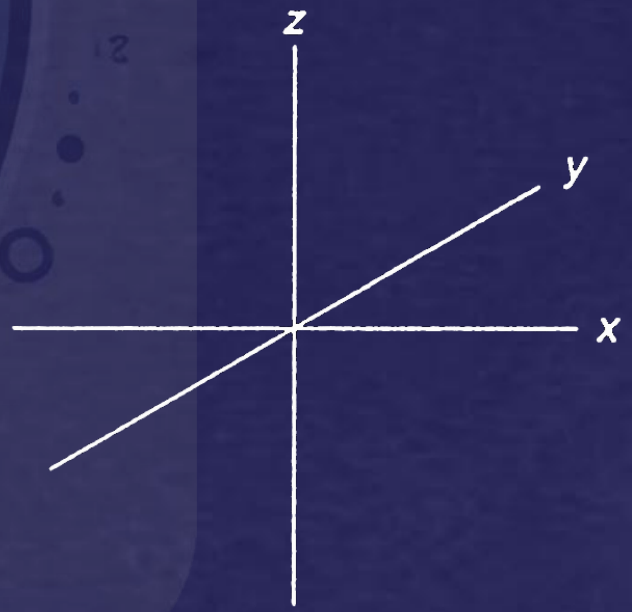
- Observations and measurements are made relative to a chosen reference frame.
- The observer's point of view affects how motion is perceived.
- Example: A car appears to move relative to a stationary observer on the street, but occupants inside the car see no motion relative to themselves.

2. Relative Motion:

- Motion is described differently depending on the frame of reference.
- Example: Two cars moving at the same speed and direction have no relative motion between them.

3. Coordinate System:

- A coordinate system is used to locate objects in a frame of reference.
- It consists of reference lines that intersect at a fixed origin.
- The **Cartesian coordinate system** uses three perpendicular axes:
    - x-axis (horizontal),
    - y-axis (horizontal),
    - z-axis (vertical, specifying up or down).



4. Inertial Frame of Reference:

- A frame where objects move at constant velocity unless acted upon by an external force.
- Observers moving with the inertial frame cannot detect whether the frame itself is in motion.

5. Key Point:

- Comparing different frames of reference helps describe motion consistently. Transforming a coordinate system to match an object's constant velocity simplifies the analysis.

---------Remember This!---------

A frame of reference defines the perspective of an observer and relies on a coordinate system to establish a fixed origin. When a frame of reference moves at a constant velocity without acceleration, it is referred to as an inertial frame of reference. This type of frame is essential for understanding motion and ensures that the laws of physics, such as Newton's first law, apply uniformly.

Scalar vs. Vector:

Quantities that are described by both **magnitude** (size) and **direction** are called **vector quantities**.

Examples include:

- **Force**
- **Velocity**
- **Displacement**
- **Weight**

In contrast, quantities that only have **magnitude** but no direction are known as **scalar quantities**, such as:

- **Mass**
- **Speed**
- **Energy**
- **Distance**

---------Remember This!---------

Vectors are distinct because they possess both **size** and **direction**, whereas scalars are described only by their magnitude.

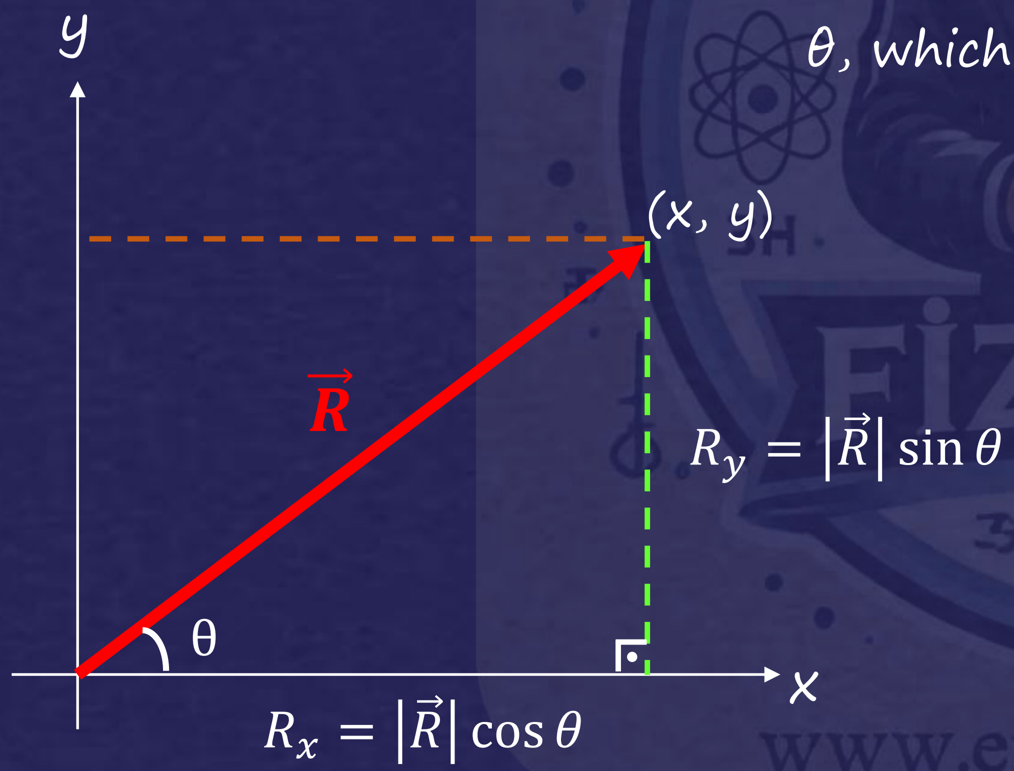
VECTORS

Another way to determine the position of an object in the Cartesian coordinate system is by using a directed line segment called a **vector**.

A vector starts at the origin and extends to a specific point in space, defining its **position vector**, often represented as $\vec{R}$

Magnitude: The length of the vector represents the straight-line distance between the origin and the point (x,y). $|\vec{R}|$

Direction: The orientation of the vector is described by the angle θ, which the vector makes with the positive x-axis.



$$R_x = R\cos\theta \qquad R_y = R\sin\theta$$

$$R^2 = R_x^2 + R_y^2$$

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

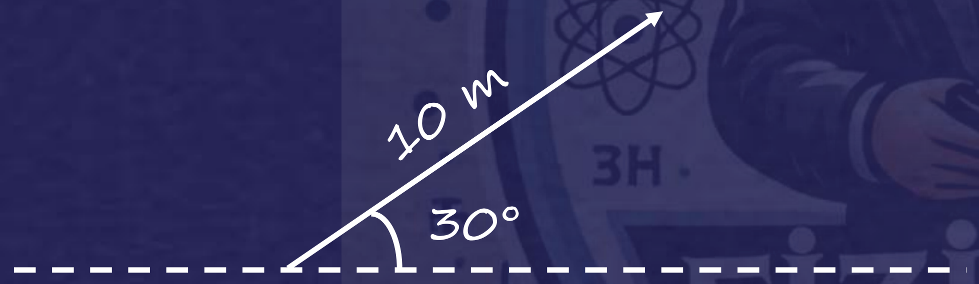
Representing Vectors in Two Forms

A vector can be described in two equivalent ways:

1.Magnitude and Direction:

This form is particularly useful for visualizing, sketching, and conducting graphical analyses.

Example: A vector with a magnitude of 10 m and a direction of 30° above the horizontal.

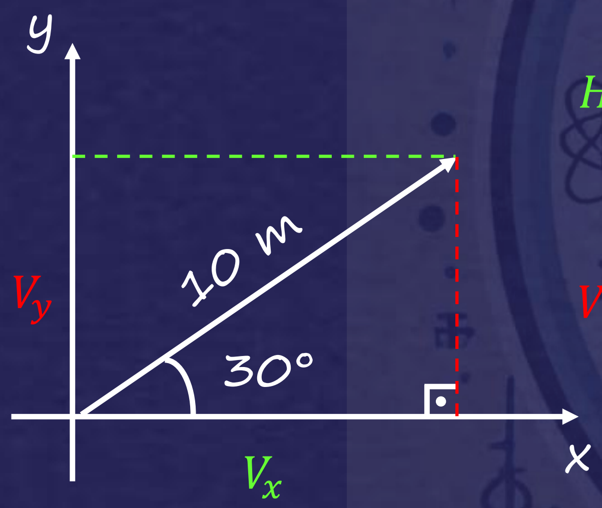


$$Magnitude\ of\ A = |\vec{A}| = 10\ m$$

$$Direction\ of\ A = \theta = 30°\ above\ the\ x\ axis.$$

2.Component Form:

1. This representation breaks the vector into its horizontal (x) and vertical (y) components, making it ideal for calculations and mathematical operations.
2. Example: A vector written as $\vec{R} = (R_x, R_y)$, where R_x and R_y are the vector's components along the x- and y- axes.



$Horizontal\ component\ of\ V = V_x = 10\cos 30° = 8.7\ m$ (8.7 m to the right)

$Vertical\ component\ of\ V = V_y = 10\sin 30° = 5\ m$ (5 m upward)

$\vec{R} = (8.7, 5)$ m

Addition of Vectors

Geometric Considerations

•Vectors can be added together to form a resultant vector. This process is essential in physics, as it allows us to combine multiple quantities with both magnitude and direction.

•Unlike adding numbers, vector addition takes both **magnitude** and **direction** into account.

•When adding two vectors, $\vec{A}$ and $\vec{B}$, the resultant vector $\vec{C}$ is expressed as:

$$\vec{C} = \vec{A} + \vec{B}$$

•The directions of $\vec{A}$ and $\vec{B}$ relative to the frame of reference must be preserved during this process.

Vector Diagram

A common method for adding vectors is by constructing a **vector diagram.** Vectors are represented by arrows with a **head** (arrow tip) and a **tail** (starting point).

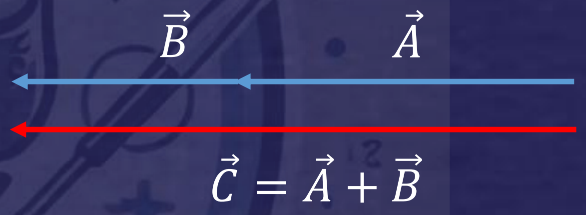
The **head-to-tail method** is often used:

- Place the tail of the second vector at the head of the first vector.
- The resultant vector is the arrow drawn from the tail of the first vector to the head of the last vector.

Example-1:

A student walks out of school and takes **3 steps west**, followed by an additional **2 steps west**. How can we represent these displacements using vectors?

1. <u>Define a Scale:</u>
    1. Let **1 step = 1 unit**.
    2. Consider west as the negative direction on a one-dimensional line (e.g., the x-axis).
2. <u>Represent the Vectors:</u>
    1. First displacement: $\vec{A} = -3\,units$
    2. Second displacement: $\vec{B} = -2\,units$



3. <u>Find the Resultant:</u>

    $|\vec{C}| = |\vec{A} + \vec{B}| = 3 + 2 = 5\,units$ $\quad \vec{C} = -5\,units$ (Since west is the negative direction, indicating 5 units west.)

4. <u>Result:</u>

    The student's total displacement is **5 units west**, which means they are now 5 units away from their starting point in the westward direction.

Example-2:

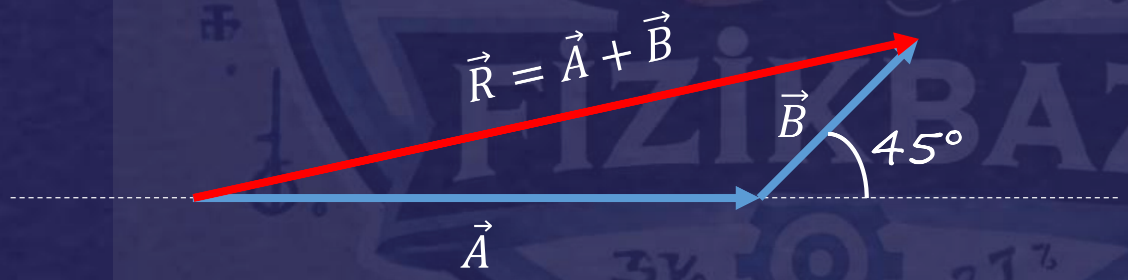
Imagine a person walks **5 blocks east** and then **2 blocks northeast** (45° north of east). How can we represent this using vectors?

1. Draw the Vector Diagram (Head-to-Tail Method):

•First vector: Draw an arrow representing **5 blocks east**, pointing to the right.

•Second vector: From the head of the first vector, draw an arrow representing **2 blocks northeast**, angled 45° above the horizontal axis.

•Resultant Vector: Draw an arrow from the tail of the first vector to the head of the second vector. This is the **resultant vector $\vec{R}$**, which represents both the magnitude and direction of the total displacement."

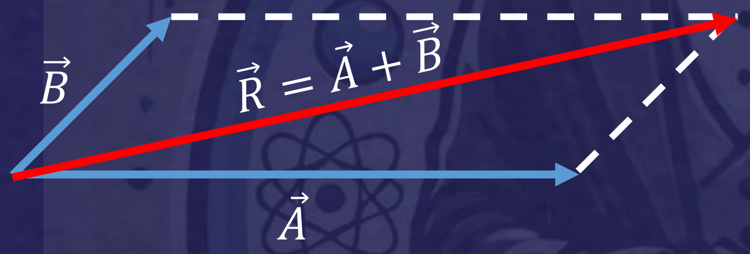


2. Alternative Method: Parallelogram Method

•Draw both vectors originating from the same point (tail-to-tail).

•Complete a parallelogram by drawing lines parallel to each vector.

•The diagonal of the parallelogram, starting from the shared tail point, represents the **resultant vector $\vec{R}$**, and its direction is consistent with the head-to-tail method.



3. Resultant Interpretation:

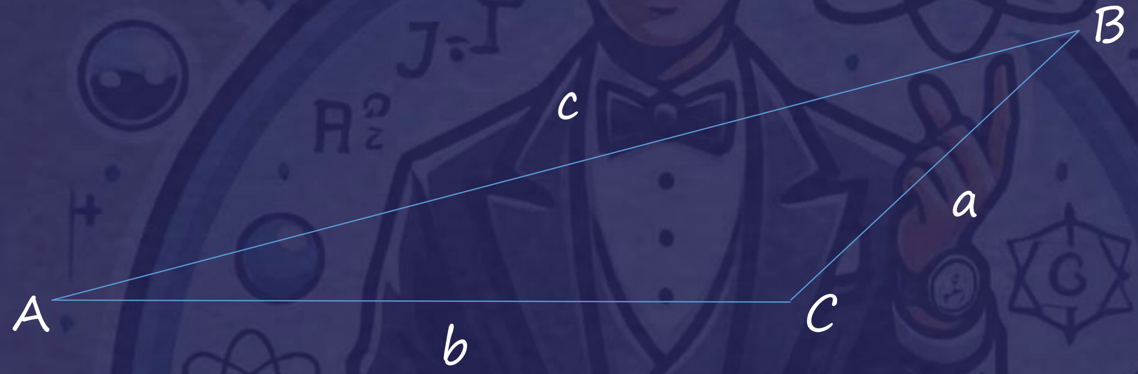
•In both methods, $\vec{R}$ represents the net displacement, with its magnitude and direction determined graphically.

---------Remember This!---------

In graphical problems, both methods lead to the same resultant vector, which ensures consistency in displacement analysis.

Algebraic Considerations

In the previous examples, we determined the resultant vector graphically. Now, let's explore how we can calculate the resultant vector's magnitude and direction algebraically, using the **Law of Cosines** and the **Law of Sines**.



Law of Cosines

The Law of Cosines relates the sides and angles of any triangle:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Where:

- a and b are the lengths of two sides of the triangle (representing the magnitudes of the original vectors).
- C is the angle between these two vectors.
- c is the side opposite angle C (the magnitude of the resultant vector).

Law of Sines

The Law of Sines relates the angles and sides of a triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

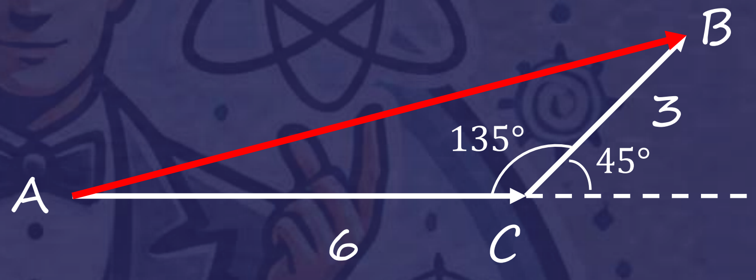
This law allows us to determine the direction of the resultant vector by calculating the angles of the triangle.

Example: Combining Two Vectors

Let's apply these laws to calculate the resultant vector's magnitude and direction using the data provided in the diagram.

Given Data:

- $a = 3$ (north-east vector at 45°),
- $b = 6$ (eastward vector),
- Angle $C = 135°$ (the angle between the vectors).



Step 1: Finding the Magnitude of the Resultant (c)

Using the Law of Cosines:

$$c^2 = a^2 + b^2 - 2abcosC$$

$$c^2 = 3^2 + 6^2 - 2 \times 3 \times 6cos(135°)$$

$$c^2 = 45 - 36cos(135°)$$

$$c^2 = 45 - 36cos(135°)$$

$$c = 8.4\ units$$

Step 2: Finding the Direction of the Resultant (A)

Using the **Law of Sines**:

$$\frac{3}{\sin A} = \frac{8.4}{\sin 135°}$$

$$\sin A = \frac{3sin135°}{8.4} = 0.25$$

$$A = \sin^{-1} 0.25 = 14.25°$$

The resultant vector is directed at **14.25° north of east.**

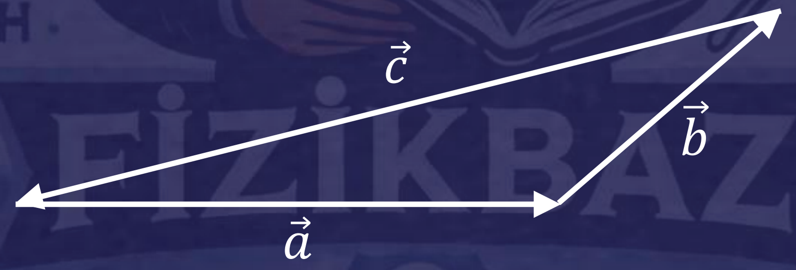
Vector Addition of Multiple Vectors

Vector addition follows the principle that vectors can be combined as long as they are arranged head to tail. When multiple vectors are added in this manner, the resultant vector represents the direct displacement from the initial to the final position.

If the resultant is zero, the vectors form a closed geometric figure, such as a triangle or polygon.

Example-1: Closed Triangle Formation: Consider three vectors a, b, and c forming a closed triangle. The equation can be expressed as:

$$\vec{a} + \vec{b} + \vec{c} = 0$$



This implies that the vectors return to the starting point, resulting in no net displacement.

Vector Subtraction

Understanding Vector Subtraction

Vector subtraction is best understood by thinking of it as adding the negative of a vector. Instead of directly subtracting a vector, we rewrite the operation by reversing the direction of the vector being subtracted.

$$\vec{R}_2 - \vec{R}_1 = \vec{R}_2 + (-\vec{R}_1)$$

A **negative vector** $(-\vec{R}_1)$ has the same magnitude as $\vec{R}_1$ but points in the opposite direction (180-degree rotation).

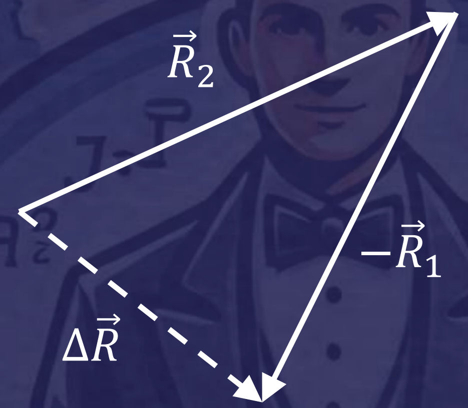
Graphical Interpretation

- The subtraction operation can be visualized by flipping $\vec{R}_1$ and adding it to $\vec{R}_2$.
- The resultant $\Delta\vec{R}$ represents the difference between the two vectors.

Example-1:

In the diagram, we see that $\vec{R}_2$ and $-\vec{R}_1$ are arranged **head-to-tail**, forming a triangle. The vector sum remains consistent:

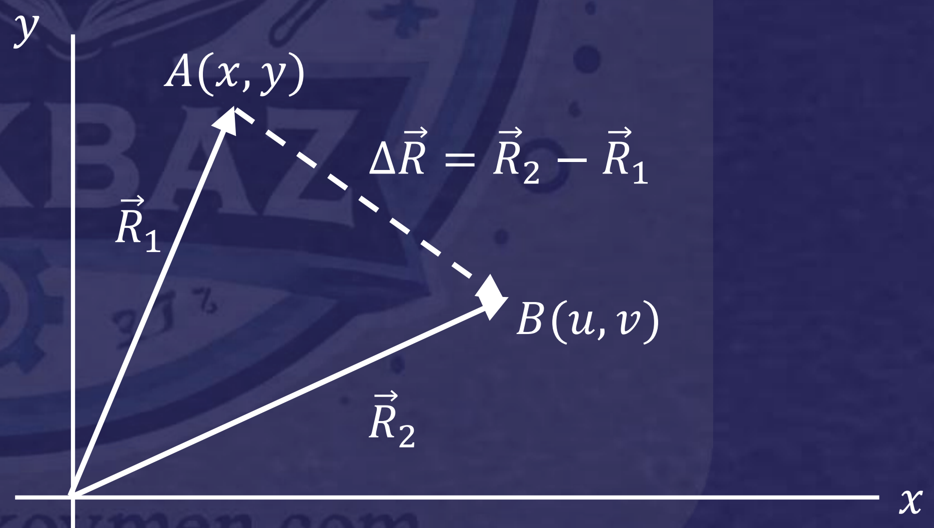
$$\vec{R}_2 + (-\vec{R}_1) = \Delta\vec{R}$$



Example-2:

In the coordinate plane, vector positions are shown explicitly. If point A represents $\vec{R}_1$ and point B represents $\vec{R}_2$, the displacement $\Delta\vec{R}$ is:

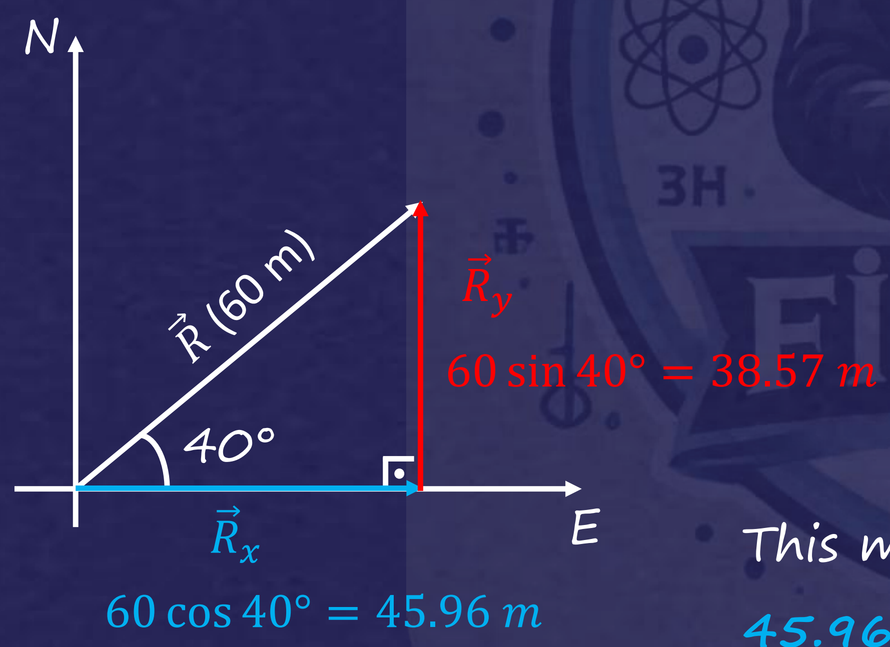
$$\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$$



Addition Methods Using the Components of Vectors

A vector in space can be broken down into two perpendicular components: one along the horizontal (x-axis) and one along the vertical (y-axis). This process is called vector resolution, and it allows us to analyze motion in different directions separately.

Consider a vector $\vec{R}$ that represents a displacement of **60 meters northeast**, making a **40° angle** with the positive x-axis.



$$R_x = |\vec{R}| \cos\theta = 60 \cos 40° = 45.96m$$

$$R_y = |\vec{R}| \sin\theta = 60 \sin 40° = 38.57m$$

This means that 60-meter displacement is equivalent to moving **45.96 meters east** and **38.57 meters north**.

Adding Vectors Using Components

When adding multiple vectors, **breaking them into x- and y-components** allows for a systematic approach. Instead of dealing with the vectors directly, we analyse their individual horizontal and vertical effects.

For two given vectors, $\vec{A}$ and $\vec{B}$:

- $\vec{A}$ has components A_x and A_y
- $\vec{B}$ has components B_x and B_y

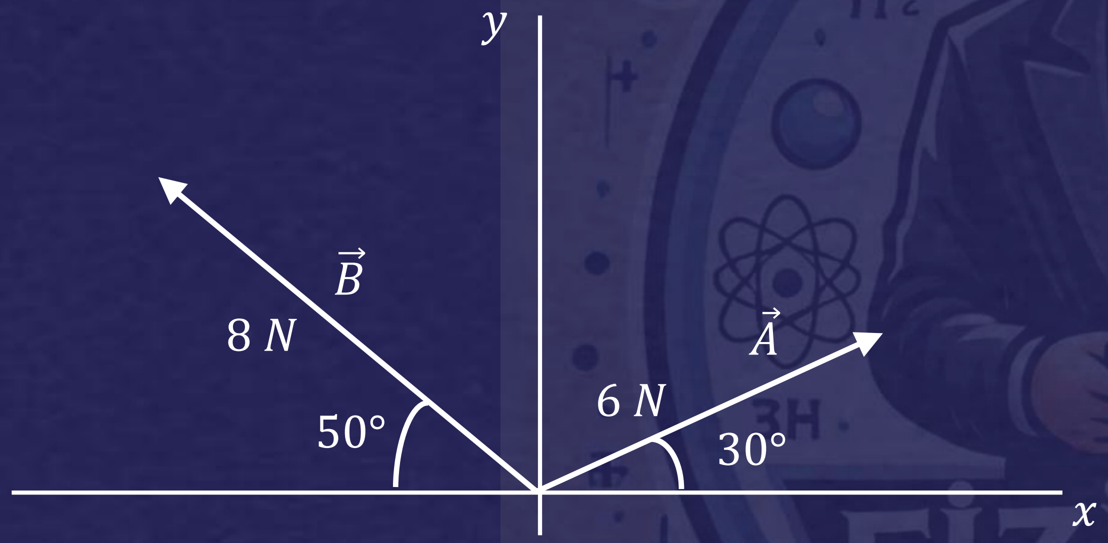
The resultant vector $\vec{C}$ is calculated using:

$C_x=A_x+B_x$ $C_y=A_y+B_y$ adding the horizontal (x) and vertical (y) components separately, we can find the net vector in a simplified manner.

Example:

Imagine two forces applied at a point:

- Force $\vec{A}$, 6 N at 30° from the positive x-axis.
- Force $\vec{B}$, 8 N at 50° from the negative x-axis.



Step-1: Resolving into Components

Using trigonometry

$A_x = 6\cos 30° = 5.20N \quad A_y = 6\sin 30° = 3.00N$

$B_x = -8\cos 50° = -5.14N \quad B_y = 8\sin 50° = 6.13N$

(Since $\vec{B}$ is positioned at an angle **relative to the negative x-axis**, its **x-component** is **negative.**)

Step-2: Resolving into Components

$$C_x = 5.20 - 5.14 = 0.06\text{N}$$

$$C_y = 3.00 + 6.13 = 9.13N$$

Step-3: Finding the Magnitude of $\vec{C}$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{(0.06)^2 + (9.13)^2} = 9.13N$$

Step-4: Finding the Direction of $\vec{C}$

The angle with the positive x- axis:

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{9.13}{0.06}\right) \cong 89.6°$$

---------Final Result---------

The magnitude of the resultant force=9.13 N

Direction=89.6° from the positive x- axis (almost vertical)

📨 We'd Love to Hear From You!

If you have any questions, suggestions, or ideas to discuss, feel free to reach out at erkan@erkankoymen.com. Whether it's a quick clarification or something more in-depth, I'd be happy to help.

Some topics are best explored through direct interaction—just let me know how I can assist you!

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