UNIT-2:KINEMATICS

Created by FİZİKBAZ for students preparing for AP Physics-1 exams.

1. Average and Instantaneous Motion

1.1 Motion and Position

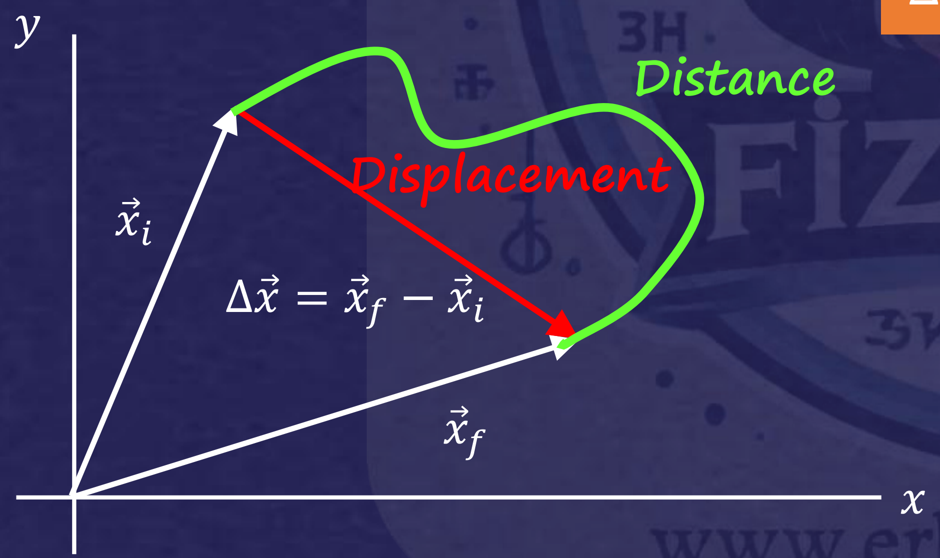
•Motion is described as the change in position of an object over time.

•To analyse motion, we use a **frame of reference**, usually a coordinate system.

•There are two important concepts in motion:

- **Distance:** The total length of the path travelled by an object. *(Scalar quantity)*
- **Displacement ($\Delta\vec{x}$):** The shortest straight-line distance between an object's initial and final position. *(Vector quantity)*

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$$



---------For Instance---------

"If a car moves 8m forward and then 3m back, distance is 11m, but displacement is 5m."

1.2 Average Velocity vs. Average Speed

•**Average velocity** is the displacement divided by the total time interval:

$$\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t}$$

- It is a **vector quantity**, meaning it has both magnitude and direction.
- SI unit: **m/s**

•**Average speed** is the total distance travelled divided by the total time:

$$s_{avg} = \frac{\textbf{total distance}}{\Delta t}$$

- It is a **scalar quantity** (only magnitude, no direction).
- Unlike velocity, speed is always positive.

---------Important---------

•If an object moves forward and then backward, its **average speed** is greater than the magnitude of **average velocity** because speed considers the total path length.
•If an object returns to its starting position, **displacement is zero**, so **average velocity = 0**, but speed is **nonzero**.

Example: A car travels 300 m east, then 100 m west in 50 s.

- Displacement: $\Delta x = 300m - 100m = 200m$
- Distance: $300m + 100m = 400m$

- Average velocity:

$$V_{avg} = \frac{\mathbf{200}}{50} = 4\ m/s$$

- Average speed:

$$s_{avg} = \frac{\mathbf{400}}{50} = 8m/s$$

--------- **Key Takeaway** ---------

Average speed ≥ Magnitude of average velocity

1.3 Instantaneous Velocity

- The instantaneous velocity of an object is its velocity at a specific moment in time.
- Unlike average velocity, which is calculated over a time interval, instantaneous velocity describes how fast an object is moving **at an** exact point.
- If velocity is constant, then instantaneous velocity = average velocity.
- If velocity is **changing**, the instantaneous velocity at a given time can be approximated by calculating the average velocity over a very short time interval.

Formula for constant velocity motion:

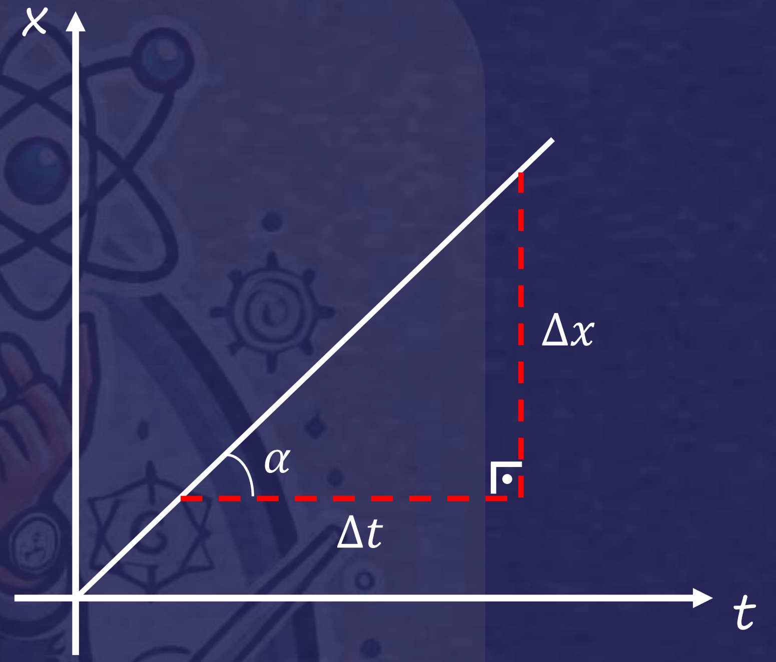
$$\vec{x} = \vec{v}\Delta t$$

1.4 Graphical Analysis of Motion

The slope of a **position-time graph** represents **velocity**.

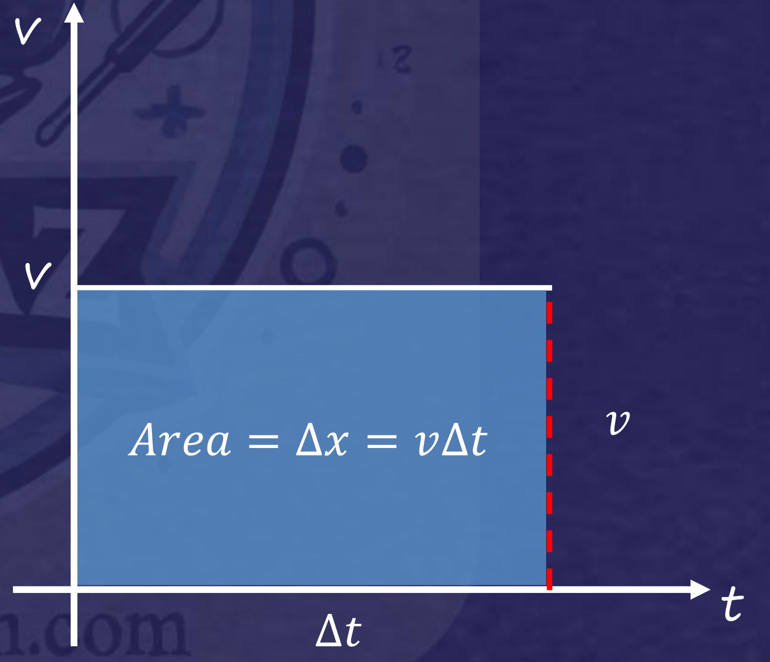
$$\boxed{Slope = tan\alpha = \frac{\Delta x}{\Delta t} = v}$$

- **Positive slope** → *Object moves forward.*
- **Negative slope** → *Object moves backward.*
- **Zero slope (flat line)** → *Object is at rest.*



The **area under a velocity-time graph** represents **displacement.**

$$\boxed{Area = \Delta x = v\Delta t}$$



2. Acceleration

What is Acceleration?

- Acceleration describes how velocity changes over time.
- An object is said to be accelerating if its speed or direction changes.
- If velocity changes **uniformly**, the object has constant acceleration.
- SI unit: m/s^2
- Acceleration is a vector quantity, meaning it has both magnitude and direction.

📌 If velocity is constant, acceleration = 0.

📌 If velocity increases, acceleration is positive.

📌 If velocity decreases, acceleration is negative (deceleration).

---------Common Misconception---------

"If an object has zero velocity, does it mean it has zero acceleration? **Not necessarily!** An object can be at rest but still accelerating (e.g., free fall at the topmost point)."

2.1 Average Acceleration

- The average acceleration over a time interval is given by:

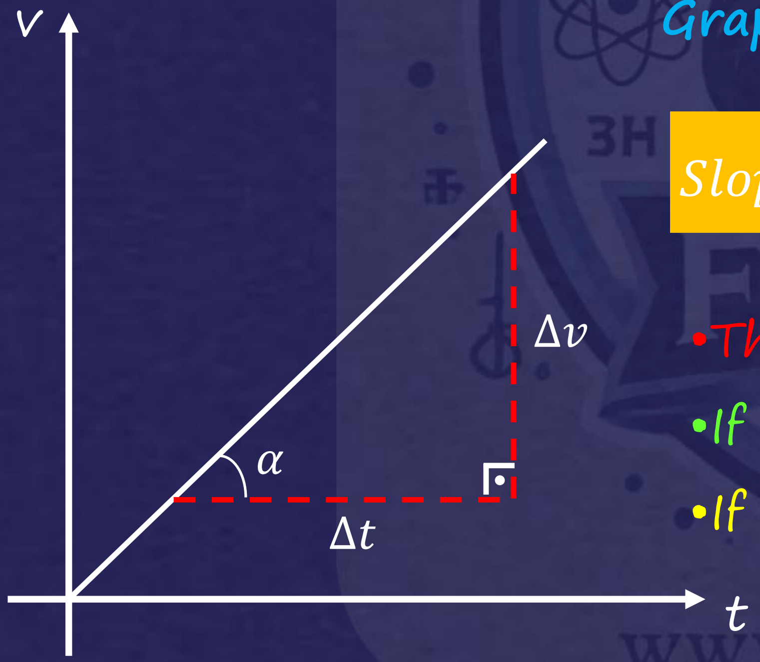
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

Where:

$\Delta\vec{v} = v_f - v_i$ is the change in velocity

Δt is the time interval

Graphical Interpretation:



$$\boxed{Slope = tan\alpha = \frac{\Delta v}{\Delta t} = a}$$

- The **slope of a velocity-time (v-t) graph represents acceleration.**
- If the graph is a **straight line**, acceleration is constant.
- If the graph is curved, acceleration is **changing**.

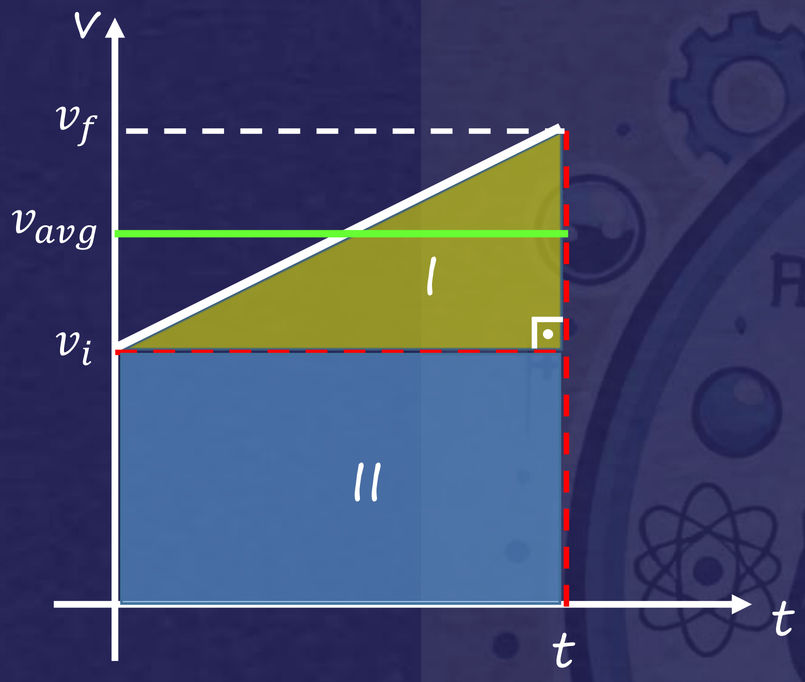
2.2 Instantaneous Acceleration:

- The instantaneous acceleration of an object is its acceleration at a specific moment in time.
- If acceleration is constant, then instantaneous acceleration = average acceleration.
- If acceleration is **changing**, the instantaneous acceleration at a given time can be approximated by calculating the average acceleration over a very short time interval.

Formula for constant acceleration motion:

$$\boxed{\Delta\vec{v} = \vec{a}\Delta t \implies v_f - v_i = at \implies v_f = v_i + at}$$

2.3 Kinematic Equations (SUVAT Equations)



The slope of the line in triangle (I)

$$Slope = a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t} \implies \boxed{v_f = v_i + at} \qquad (1)$$

The area of the triangle (I) $\implies A_1 = \frac{(v_f - v_i)t}{2} = \frac{(v_f - v_i)t}{2}\frac{t}{t}$

$$A_1 = \frac{(v_f - v_i)t^2}{2t} = \frac{1}{2}\frac{(v_f - v_i)}{t}t^2$$

$$A_1 = \frac{1}{2}at^2$$

The area of the rectangle (II) $\implies A_2 = v_i t$

The total area under the graph=Total displacement $\implies \boxed{x = v_i t + \frac{1}{2}at^2} \qquad (2)$

From the graph $\implies \boxed{v_{avg} = \frac{v_i + v_f}{2}} \qquad (3)$

2.3 Kinematic Equations (SUVAT Equations)-2

$$v_f = v_i + at \implies t = \frac{v_f - v_i}{a}$$

$$x = v_i t + \frac{1}{2}at^2 = v_i \frac{v_f - v_i}{a} + \frac{1}{2}a\left(\frac{v_f - v_i}{a}\right)^2$$

$$x = v_i \frac{v_f - v_i}{a} + \frac{1}{2}a\left(\frac{v_f - v_i}{a}\right)^2 = \frac{v_f^2 - v_i^2}{2a} \implies 2ax = v_f^2 - v_i^2 \quad \text{or} \quad \boxed{v_f^2 = v_i^2 + 2ax} \qquad (4)$$

---------SUVAT Equations---------

$$\boxed{v_f = v_i + at} \qquad \boxed{v_{avg} = \frac{v_i + v_f}{2}} \qquad \boxed{x = v_i t + \frac{1}{2}at^2} \qquad \boxed{v_f^2 = v_i^2 + 2ax}$$

Example: A particle accelerates from rest at a uniform rate of 6 m/s² for a distance of 400 m. How fast is the particle going at that time? How long did it take for the particle to reach that velocity?

$$\boxed{v_f^2 = v_i^2 + 2ax}$$

$v_i = 0 \quad \Longrightarrow \quad v_f^2 = 2ax \qquad v_f^2 = 2 \times 6 \times 400 = 4800 \quad \Longrightarrow \quad v_f = 69.3 \text{ m/s}$

To find time:

$v_f = v_i + at \quad \Longrightarrow \quad t = \dfrac{69.3}{6} = 11.6\ s$

Example: A high-speed train is traveling at 150 m/s when it begins to slow down due to constant braking. If the train experiences a uniform deceleration of -3 m/s², how far will it travel before coming to a complete stop?

We do not know the time needed to stop, but we do know that the final velocity is zero. If we use the formula:

$$\boxed{v_f^2 = v_i^2 + 2ax}$$

$$0 = 150^2 + 2 \times (-3)x$$

$$x = \frac{150^2}{6} = 3750m$$

3.Accelerated Motion Due to Gravity

3.1 Understanding Motion Under Gravity

- Gravity provides a constant downward acceleration.
- On Earth, this acceleration is $g = 9.8\ m/s^2$
- An object moving upward slows down due to gravity.
- An object moving downward speeds up due to gravity.
- Gravity affects both vertical and projectile motion.

---------Don't Forget---------

Students often forget that there is still gravitational acceleration even at the top of a projectile's path! The acceleration vector never changes during free fall (including during projectile motion).

3.2 Kinematic Equations

To describe the motion of objects under gravity, we use the kinematic equations, replacing acceleration with **–g**. These equations describe motion under uniform acceleration, specifically under gravity:

1.Velocity-time relation:

$$v_f = v_i - gt$$

2.Displacement-time relation:

$$y = v_i t - \frac{1}{2} g t^2$$

3.Velocity-displacement relation (no time involved):

$$v_f^2 = v_i^2 - 2gy$$

Example: A rock is thrown vertically upwards with an initial velocity of **20 m/s**. How high does it go? How long does it take to reach the peak?

Step 1: Find Maximum Height

At maximum height, the final velocity is **0**.

$$\boxed{v_f^2 = v_i^2 - 2gy} \implies 0 = 20^2 - 2 \times 9.8 \times y$$

$$y = \frac{400}{19.6} = 20.4m$$

Step 2: Find Time to Reach Peak

$$\boxed{v_f = v_i - gt} \implies 0 = 20 - 9.8t \implies t = \frac{20}{9.8} = 2.04s$$

Thus, the rock reaches a **maximum height of 20.4 m** and takes **2.04 seconds** to reach the peak.

4. Graphical Analysis of Motion

Graphical analysis is a powerful tool to understand motion. By examining motion graphs, we can extract key information about an object's velocity, acceleration, and displacement over time. Particularly for uniformly accelerated motion, graphs provide a visual representation of how motion evolves.

4.1 Representing Motion with Graphs

The motion of an object can be analysed using three fundamental graphs:

1. Displacement-Time Graph (x-t)

- The slope of this graph represents the object's velocity.
- A linear graph indicates constant velocity.
- A parabolic graph suggests accelerated motion.

2. Velocity-Time Graph (v-t)

The slope of this graph represents the object's acceleration (a).

A horizontal line means constant velocity (zero acceleration).

A line with a positive slope represents increasing velocity (acceleration).

A line with a negative slope represents decreasing velocity (deceleration).

The area under the curve represents the total displacement.

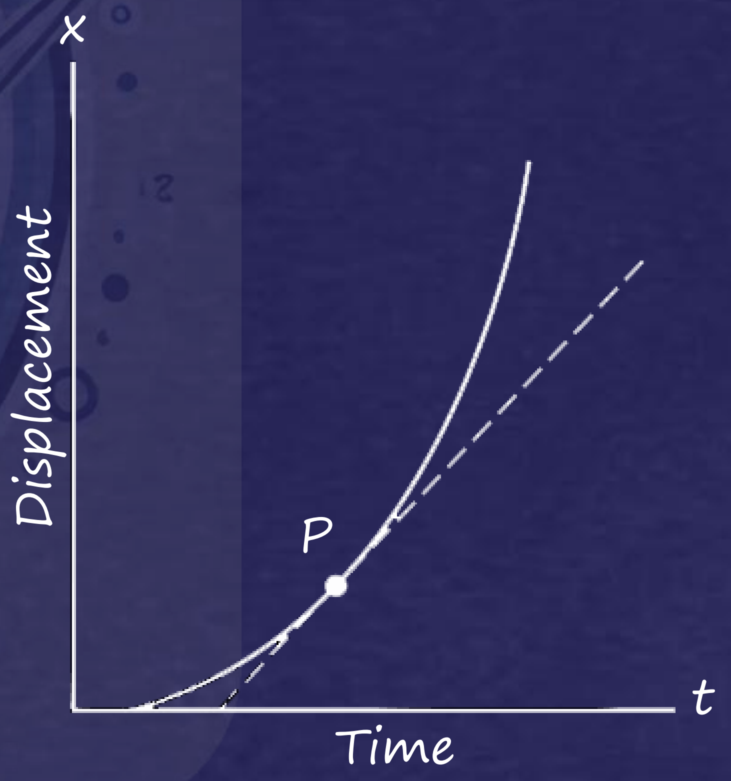
3. Acceleration-Time Graph (a-t)

1. A horizontal line means constant acceleration.
2. The area under the curve gives the change in velocity (Δv).

4.2 Displacement-Time Graph and Instantaneous Velocity

For uniformly accelerated motion, the displacement-time graph is parabolic. The slope at any point represents the instantaneous velocity.

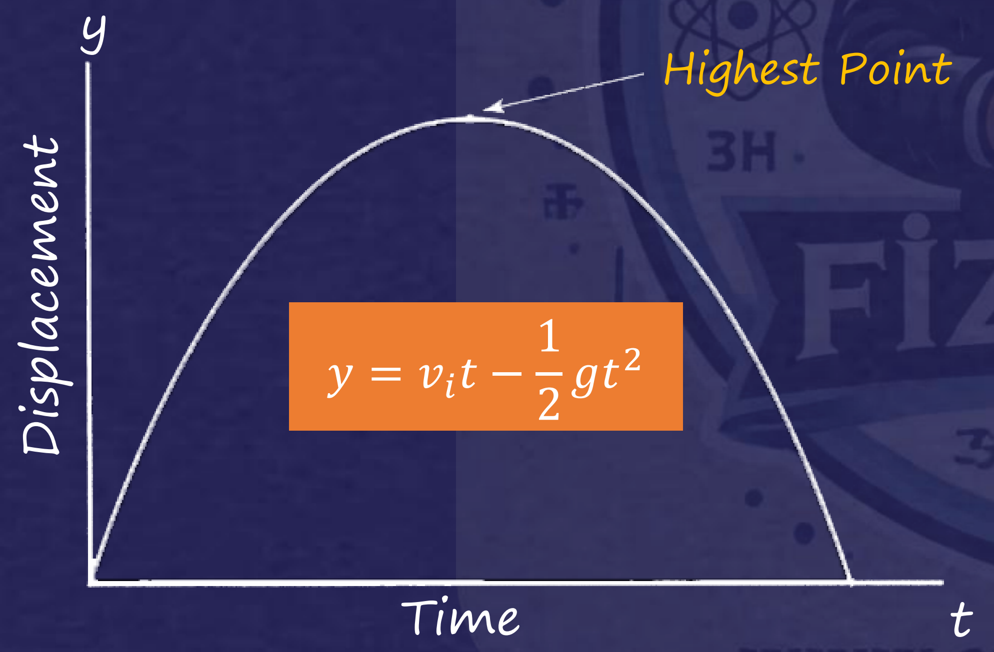
To determine the velocity at a specific point, we draw a tangent line at that point and measure its slope.

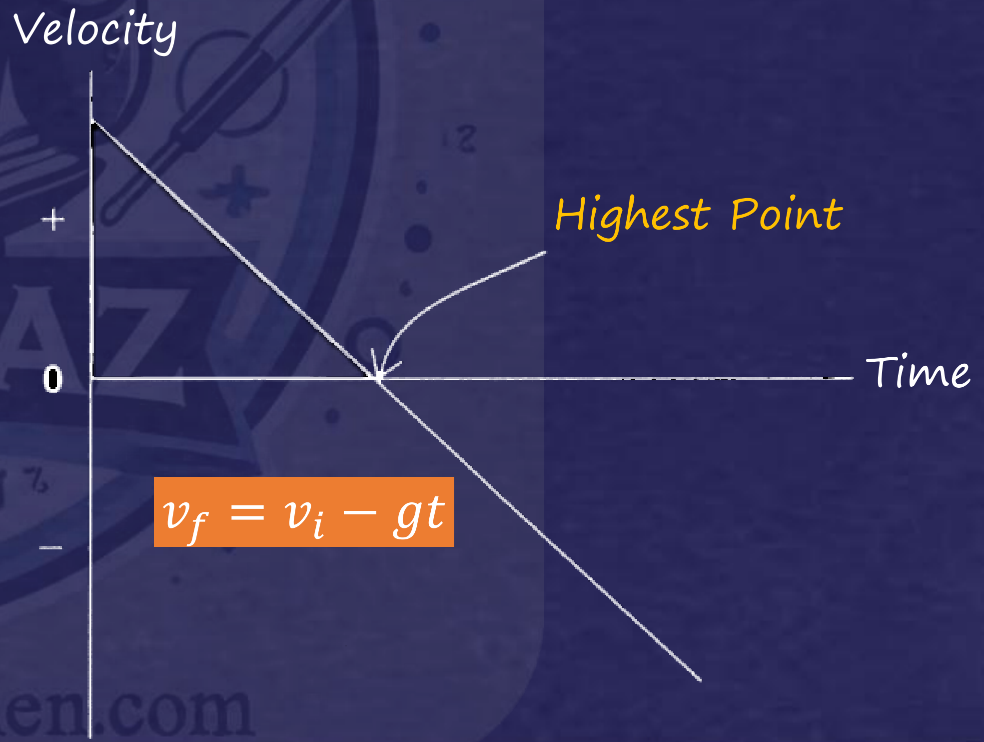


4.3 Understanding Changing Velocity with Graphs

An object's velocity can increase or decrease over time, and this is reflected in the velocity-time graph.

- If an object **accelerates**, the velocity-time graph has a **positive slope**.
- If an object **decelerates**, the velocity-time graph has a **negative slope**.
- If an object **moves upward**, velocity decreases until it reaches **zero** at the highest point.
- As the object **falls back down**, velocity becomes **negative**, indicating downward motion.





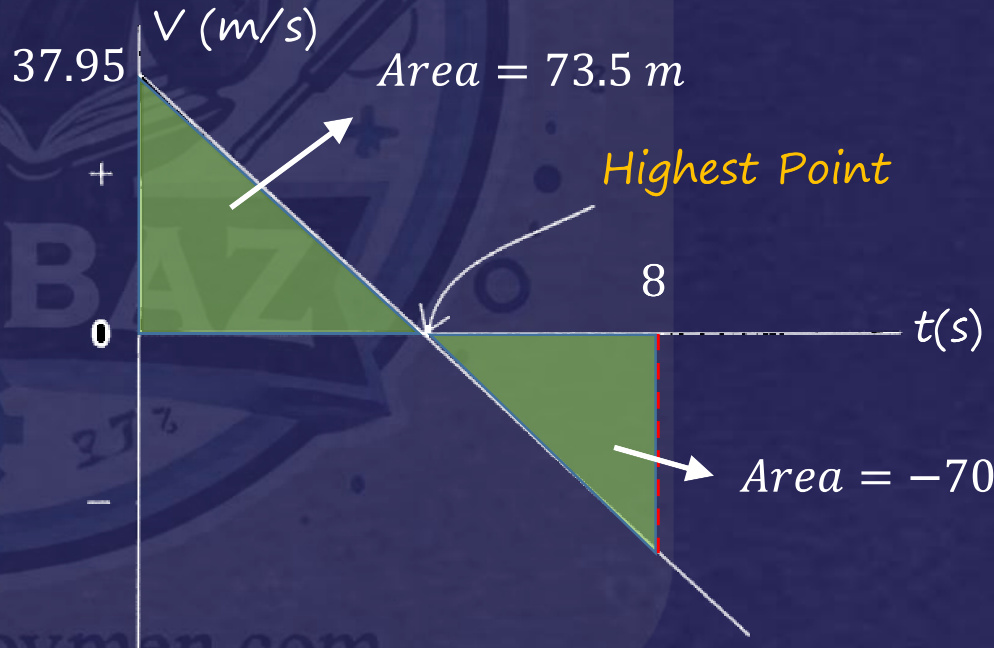
Example: A soccer player kicks a ball straight up into the air. The ball remains in the air for **8 seconds** before being caught by a teammate **3 meters above the ground**.

To what maximum height did the ball reach?

Step 1: Find the Initial Velocity

Since the ball follows a symmetrical motion, the time to reach the highest point is half of the total time in the air.

$$\boxed{y = v_i t - \frac{1}{2} g t^2} \implies 3 = v_i \times 8 - \frac{1}{2} \times 9.8 \times (8)^2 \implies 8v_i = 313.6 \implies v_i = 37.95\ m/s$$



Step 2: Find Maximum Height

Using:

$$\boxed{v_f^2 = v_i^2 - 2gy}$$

(At the highest point, v_f=0)

$$0 = 37.95^2 - 2 \times 9.8y$$

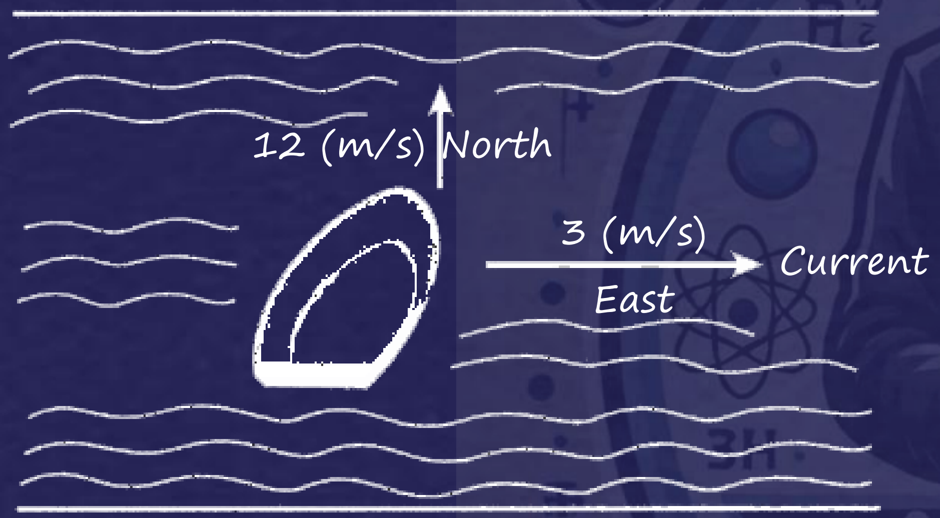
$$y = 73.5m$$

5. Relative Motion

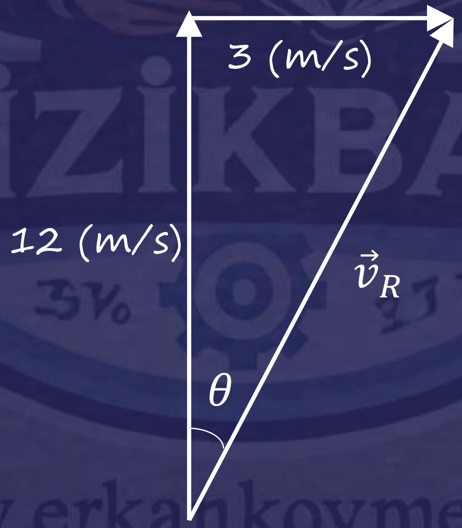
Motion is always described relative to a specific reference frame. Just as displacement and velocity are vector quantities, their values depend on the observer's perspective. For instance, when we say a train is moving at **30 m/s**, this means **30 m/s relative to the ground**. However, if an observer is on another train moving parallel at **25 m/s**, the train will appear to be moving at **5 m/s** relative to this second observer.

To analyze relative motion, we use **vector addition** to combine different velocities measured from different reference frames.

Example: Consider a boat trying to cross a river where the water flows at **3 m/s eastward.** The boat, relative to the water, moves at **12 m/s northward**. From the perspective of an observer standing on the riverbank, the boat's effective velocity is different from what is measured by someone inside the boat.



To determine the **resultant velocity** (the actual velocity of the boat relative to the riverbank), we use **vector addition**. Since the motion consists of two perpendicular components, we apply the **Pythagorean theorem**:



$$v_R = \sqrt{12^2 + 3^2} = \sqrt{153} \cong 12.4 m/s$$

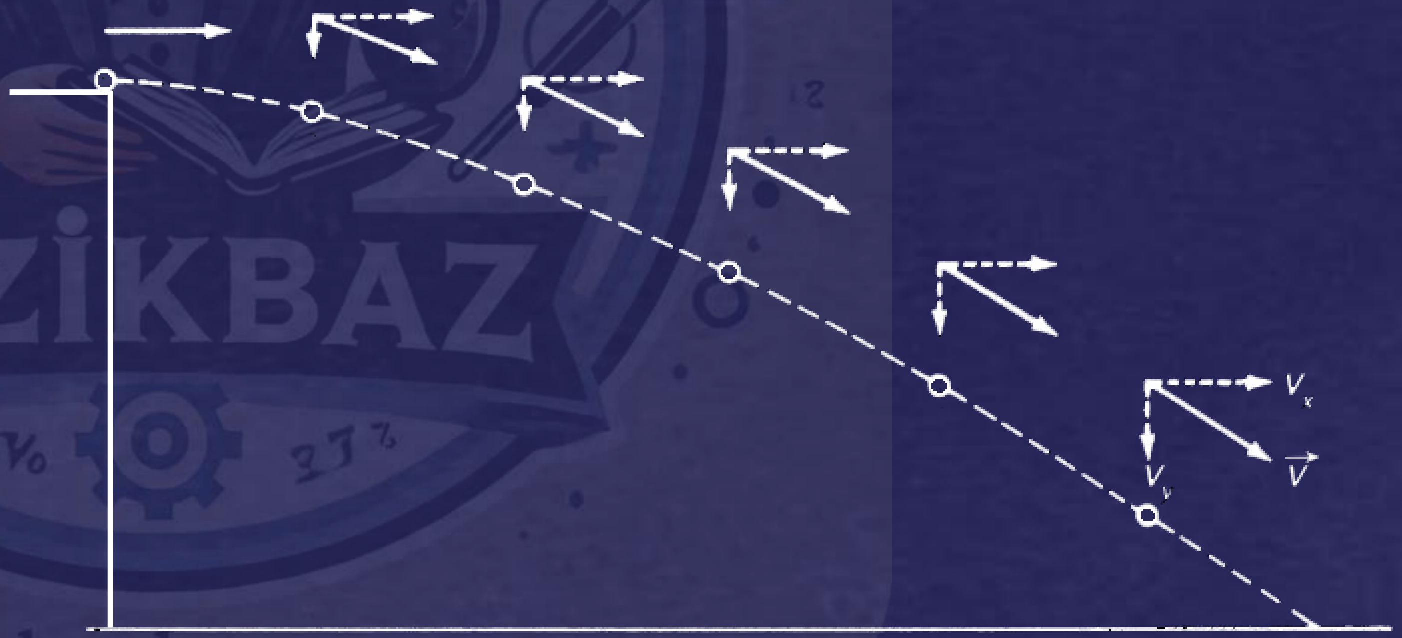
$$\theta = \tan^{-1}\left(\frac{3}{12}\right) = 14°$$

6. Horizontally Launched Projectiles

When an object is launched horizontally from an elevated surface, it does not drop straight down. Instead, it follows a curved trajectory due to the combination of two independent motions:

- **Horizontal Motion**: The object maintains a constant velocity, as there is no horizontal acceleration (assuming air resistance is negligible).
- **Vertical Motion:** The object experiences free fall due to gravity, which causes it to accelerate downward at $g=9.8\ m/s^2$

These two motions occur **simultaneously and independently**, meaning the horizontal motion does not influence the vertical motion, and vice versa. As a result, the path followed by the projectile is a **parabolic trajectory**, as first demonstrated by Galileo.



6.1 Mathematical Representation

1. Vertical Motion (Free Fall)

Since the object is initially moving only in the horizontal direction, the vertical component of the initial velocity is **zero** ($v_{iy} = 0$). The displacement in the vertical direction is given by the free-fall equation:

$$\Delta y = v_{iy}t + \frac{1}{2}gt^2$$

Since $v_{iy} = 0$, this simplifies to:

$$\Delta y = \frac{1}{2}gt^2$$

2. Horizontal Motion

The horizontal displacement Δx, also called the **range**, is given by:

$$\Delta x = v_{ix}t$$

Since there is no horizontal acceleration ($a_x = 0$), the horizontal velocity remains **constant**.

3. Derivation of Trajectory Equation

Since the time variable ttt is the same for both motions, we can solve for t from the horizontal equation:

$$t = \frac{\Delta x}{v_{ix}}$$

Substituting this into the vertical motion equation:

$$\Delta y = \frac{1}{2} g \left(\frac{\Delta x}{v_{ix}}\right)^2$$

Rearranging:

$$\Delta y = g \frac{(\Delta x)^2}{2v_{ix}^2}$$

This is the **trajectory equation** of a horizontally launched projectile, which describes a parabolic path.

Example:

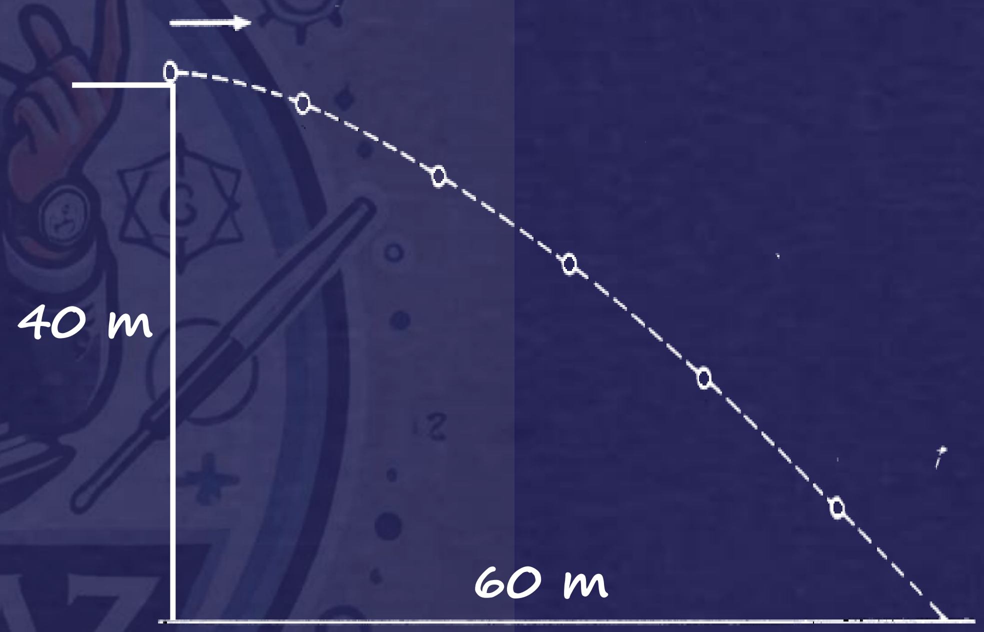
A ball is launched horizontally from a table that is **40 m** high. If the ball lands **60 m** away from the base of the table, what was its initial horizontal velocity?

Step 1 Find Time of Flight:

Using the free-fall equation:

$$\Delta y = \frac{1}{2} g t^2$$

$$40 = \frac{1}{2} \times 9.8 t^2 \implies t^2 = \frac{80}{9.8} = 8.16 \implies t = 2.86 s$$



Step 2 Find Horizontal Velocity:

Since the horizontal velocity remains constant

$$\Delta x = v_{ix} t \implies 60 = v_{ix}(2.86) \implies v_{ix} = 20.98 m/s$$

7.Projectiles Launched at an Angle

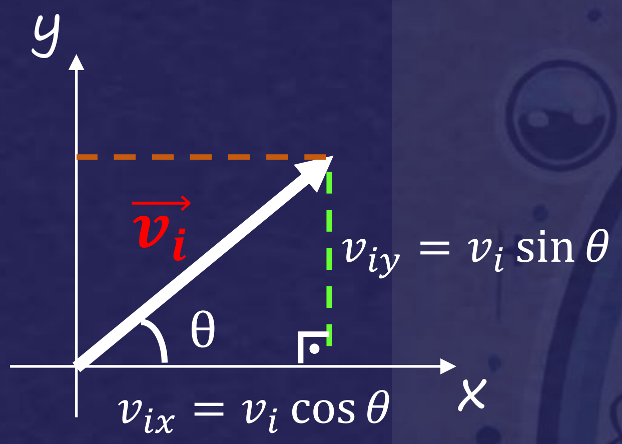
When an object is launched into the air at an angle, its motion can be analyzed by breaking it into two independent components: **horizontal motion** and **vertical motion**.

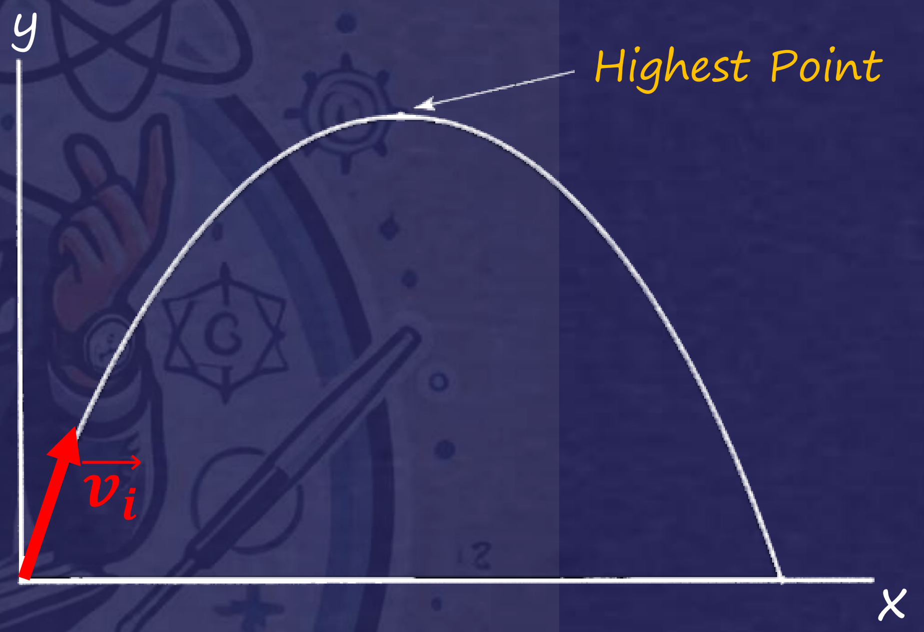
- **Horizontal Motion:** The object moves with a constant velocity since there is no horizontal acceleration (assuming air resistance is negligible).
- **Vertical Motion:** The object is influenced by gravity, causing it to decelerate while ascending and accelerate while descending, with a constant acceleration of $g = 9.8\ m/s^2$ downward.

These two components of motion are **independent** of each other, meaning the horizontal motion does not affect the vertical motion, and vice versa. The combination of both motions results in a **parabolic trajectory**, as demonstrated by Galileo.

7.1 Breaking Down the Initial Velocity

The initial velocity v_i can be decomposed into its horizontal and vertical components:





$$v_{ix} = v_i \cos\theta$$

$$v_{iy} = v_i \sin\theta$$

where:

- v_{ix} remains constant throughout the motion,
- v_{iy} decreases until it reaches zero at the highest point, then increases in the downward direction.

7.2 Finding Time to Reach Maximum Height

At the highest point, the vertical velocity is zero.

Using the kinematic equation:

$$\boxed{v_f = v_i + at}$$

$$v_y = v_{iy} - g t_{up}$$

Solving for t_{up}:

$$\boxed{t_{up} = \frac{v_{iy}}{g}}$$

Since the projectile follows a symmetric path, the total flight time is:

$$t_{total} = 2t_{up} = \frac{2v_{iy}}{g}$$

7.3 Maximum Height

The maximum height can be found using the kinematic equation:

$$y = v_i t + \frac{1}{2} a t^2$$

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

Substituting t_{up}:

$$y_{max} = \frac{v_{iy}^2}{2g}$$

SUVAT Equations

$$v_f = v_i + at$$

$$v_{avg} = \frac{v_i + v_f}{2}$$

$$x = v_i t + \frac{1}{2} a t^2$$

$$v_f^2 = v_i^2 + 2ax$$

7.4 Range (Horizontal Distance Traveled)

The range of a projectile is given by:

$$R = v_{ix} t_{total}$$

$$R = (v_i \cos\theta) t_{total}$$

Substituting $t_{total} = \frac{2v_{iy}}{g}$:

$$R = (v_i \cos\theta)\frac{2v_{iy}}{g} = \frac{2(v_i \cos\theta)(v_i \sin\theta)}{g}$$

$$\boxed{R = \frac{v_i^2 \sin 2\theta}{g}}$$

This shows that the maximum range occurs when $\theta = 45°$ (assuming launch and landing at the same height).

Example: A soccer player kicks a ball at an angle of **35°** with an initial speed of **28 m/s.**

a) What is the **maximum height** the ball reaches?

b) What is the **total time of flight?**

c) What is the **range** of the projectile?

Solution

Step 1: Find the Vertical Velocity Component

$$v_{iy} = v_i \sin\theta$$

$$v_{iy} = 28 \sin 35°$$

$$v_{iy} = 16.06\ m/s$$

Step 2: Find Maximum Height

$$y_{max} = \frac{v_{iy}^2}{2g} = \frac{(16.06)^2}{2 \times 9.8} = 13.2m$$

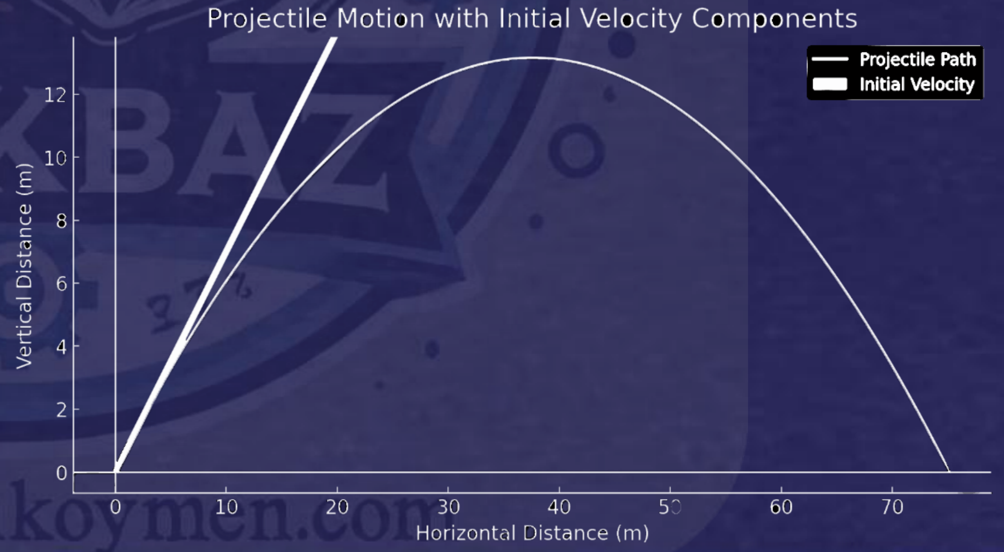
Step 3: Find Total Time of Flight

$$t_{total} = 2t_{up} = \frac{2v_{iy}}{g} = \frac{2 \times 16.06}{9.8} = 3.28s$$

Step 4: Find the range

$$v_{ix} = v_i \cos\theta = 28 \cos 35 = 22.94\ m/s$$

$$R = 22.94 \times 3.28 = 75.3\ m$$



8. Uniform Circular Motion

8.1 Understanding Uniform Circular Motion

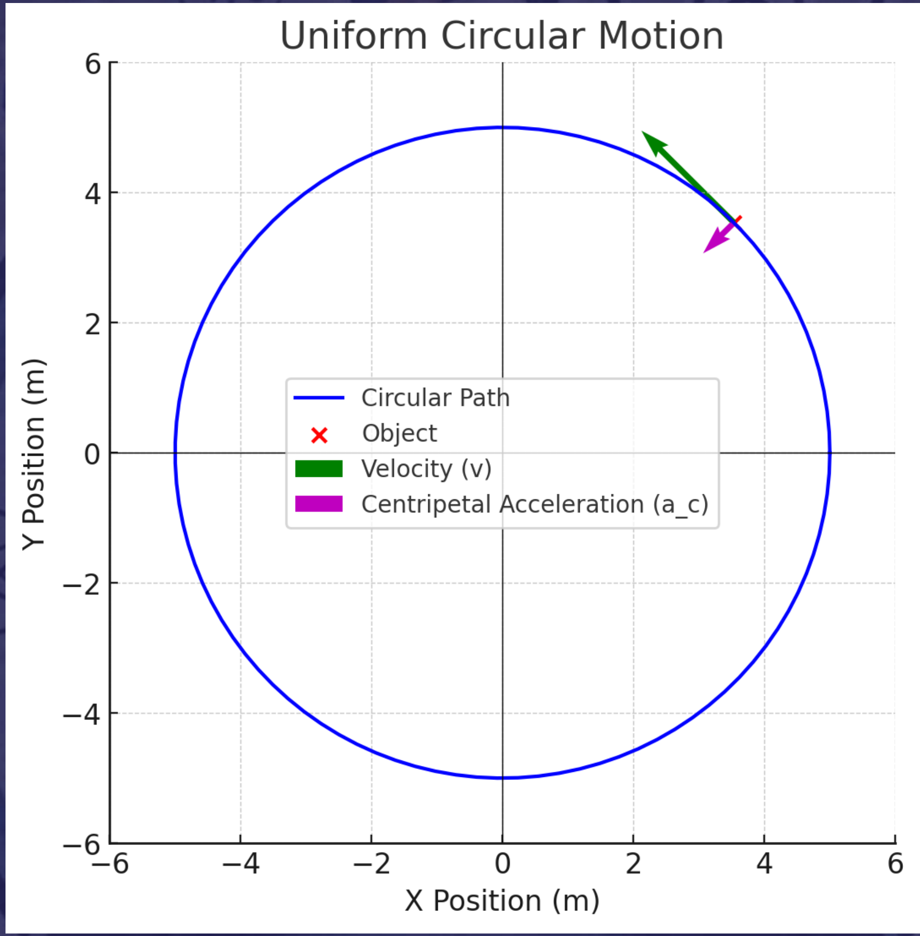
When an object moves in a circular path at a constant speed, it undergoes **uniform circular motion.** Even though the speed remains constant, the object's **velocity** is continuously changing because its **direction** is changing. Since velocity is a vector quantity, a change in direction means a change in velocity, which implies acceleration.

The acceleration that keeps the object moving in a circular path is called **centripetal acceleration**.

This acceleration is always directed **toward the centre** of the circle.

The diagram below illustrates an object moving in a circular path:

Diagram: Object in Circular Motion with Velocity and Acceleration Vectors



- The green arrow represents the velocity vector $\vec{v}$, which is always **tangent** to the circular path.
- The purple arrow represents the centripetal acceleration $\vec{a}$, which always points toward the centre of the circle.

8.2 Mathematical Representation

The magnitude of the centripetal acceleration is given by:

$$a_c = \frac{v^2}{r}$$

where:

- v^2 is the linear speed of the object,
- r is the radius of the circular path.

From the definition of speed in circular motion:

$$v = \frac{2\pi r}{T}$$

where:

- T is the period (time for one complete revolution),
- $2\pi r$ is the circumference of the circle.

By substituting this equation into the centripetal acceleration formula, we get:

$$a_c = \frac{4\pi^2 r}{T^2}$$

8.3 Period and Frequency in Circular Motion

- The **period** (T) is the time required for one complete revolution.
- The **frequency** (f) is the number of revolutions per second and is related to the period by:

$$T = \frac{1}{f}$$

Another useful concept is angular velocity (ω), which describes how fast the object rotates:

$$\omega = 2\pi f$$

where:

- T is the period (time for one complete revolution),
- $2\pi r$ is the circumference of the circle.

By substituting this equation into the centripetal acceleration formula, we get:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Example:

A 4-kg object is moving in uniform circular motion with a speed of 12 m/s in a circle of radius 3 m. Calculate the centripetal acceleration.

Solution:

Using the formula:

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{12^2}{3} = \frac{144}{3} = 48\, m/s^2$$

📨 We'd Love to Hear From You!

If you have any questions, suggestions, or ideas to discuss, feel free to reach out at erkan@erkankoymen.com. Whether it's a quick clarification or something more in-depth, I'd be happy to help.

Some topics are best explored through direct interaction— just let me know how I can assist you!

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